Math 466 - Fall 2020 - Homework 2

Due: October 6, 2020
For all of the problems show your steps and explain your answers.

Problem 1

Suppose that $\hat{\theta}$ is an estimator for a parameter θ and $\mathbb{E}[\hat{\theta}] = a\theta + b$ for some nonzero constants a and b.

a) In terms of a, b, and θ, what is $B(\hat{\theta})$?

b) Find a function of $\hat{\theta}$-say, $\hat{\theta}^*$-that is an unbiased estimator for θ.

Problem 2

Suppose that Y_1, Y_2, Y_3 denote a random sample from an exponential distribution with density function

$$f(y) = \begin{cases} \frac{1}{\theta}e^{-\frac{y}{\theta}}, & y \geq 0, \\ 0, & \text{otherwsise.} \end{cases}$$

Consider the following five estimators of θ: $\hat{\theta}_1 = Y_1$, $\hat{\theta}_2 = \frac{Y_1+Y_2}{2}$, $\hat{\theta}_3 = \frac{Y_1+2Y_2}{3}$, $\hat{\theta}_4 = \min(Y_1, Y_2, Y_3)$ and $\hat{\theta}_5 = \overline{Y}$.

a) Which of these estimators are unbiased?

b) Among the unbiased estimators, which has the smallest variance?

Problem 3

In a study to compare the perceived effects of two pain relievers, 200 randomly selected adults were given the first pain reliever, and 93% indicated appreciable pain relief. Of the 450 individuals given the other pain reliever, 96% indicated experiencing appreciable relief.

a) Give an estimate for the difference in the proportions of all adults who would indicate perceived pain relief after taking the two pain relievers. Provide a bound on the error of estimation.

b) Based on your answer to part (a), is there evidence that proportions experiencing relief differ for those who take the two pain relievers? Why?

Problem 4

Suppose that Y is normally distributed with mean 0 and unknown variance σ^2. Then $\frac{Y^2}{\sigma^2}$ has a χ^2 distribution with 1 degrees of freedom. Use the pivotal quantity $\frac{Y^2}{\sigma^2}$ to find a

a) 95% confidence interval for σ^2.

b) 95% upper confidence limit for σ^2.

c) 95% lower confidence limit for σ^2.

Problem 5

The administrators for a hospital wished to estimate the average number of days required for inpatient treatment of patients between the ages of 25 and 34. A random sample of 500 hospital patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95% confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

Problem 6

One suggested method for solving the electric-power shortage in a region involves constructing floating nuclear power plants a few miles offshore in the ocean. Concern about the possibility of a ship collision with the floating (but anchored) plant has raised the need for an estimate of the density of ship traffic in the area. The number of ships passing within 10 miles of the proposed power-plant location per day, recorded for $n = 60$ days during July and August, possessed a sample mean and variance of $\overline{y} = 7.2$ and $s^2 = 8.8$.

a) Find a 95% confidence interval for the mean number of ships passing within 10 miles of the proposed power-plant location during a 1-day time period.

b) The density of ship traffic was expected to decrease during the winter months. A sample of $n = 90$ daily recordings of ship sightings for December, January, and February yielded a mean and variance of $\overline{y} = 4.7$ and $\sigma^2 = 4.9$. Find a 90% confidence interval for the difference in mean density of ship traffic between the summer and winter months.

c) What is the population associated with your estimate in part (b)? What could be wrong with the sampling procedure for parts (a) and (b)?

Problem 7

Let Y be a binomial random variable with parameter p. Find the sample size necessary to estimate p to within 0.05 with probability 0.95 in the following situations:

a) If p is thought to be approximately 0.9

b) If no information about p is known (use $p = 0.5$ in estimating the variance of $\hat{p}$).

Problem 8

Suppose that you want to estimate the mean pH of rainfalls in an area that suffers from heavy pollution due to the discharge of smoke from a power plant. Assume that σ is in the neighborhood of 0.5 pH and that you want your estimate to lie within 0.1 of μ with probability near 0.95. Approximately how many rainfalls must be included in your sample (one pH reading per rainfall)? Would it be valid to select all of your water specimens from a single rainfall? Explain.

Problem 9

Organic chemists often purify organic compounds by a method known as fractional crystallization. An experimenter wanted to prepare and purify 4.85g of aniline. Ten 4.85-gram specimens of aniline were prepared and purified to produce acetanilide. The following dry yields were obtained:

$$3.85, 3.88, 3.90, 3.62, 3.72, 3.80, 3.85, 3.36, 4.01, 3.82.$$

Construct a 95% confidence interval for the mean number of grams of acetanilide that can be recovered from 4.85 grams of aniline.

Problem 10

Two new drugs were given to patients with hypertension. The first drug lowered the blood pressure of 16 patients an average of 11 points, with a standard deviation of 6 points. The second drug lowered the blood pressure of 20 other patients an average of 12 points, with a standard deviation of 8 points. Determine a 95% confidence interval for the difference in the mean reductions in blood pressure, assuming that the measurements are normally distributed with equal variances.