Math 466 - Fall 2020 - Homework 1

Due: September 17, 2020

For all of the problems show your steps and explain your answers.

Problem 1 Suppose that $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ are independent random samples, with the variables X_i normally distributed with mean μ_1 and variance σ_1^2 and the variables Y_i normally distributed with mean μ_2 and variance σ_2^2.

- Find $\mathbb{E}(\overline{X} - \overline{Y})$.
- Find $V(\overline{X} - \overline{Y})$.
- Suppose that $\sigma_1^2 = 2$, $\sigma_2^2 = 2.5$, and $m = n$. Find the sample sizes so that $(\overline{X} - \overline{Y})$ will be within 1 unit of $\mu_1 - \mu_2$ with probability 0.95.

Problem 2 A forester studying the effects of fertilization on certain pine forests in the Southeast is interested in estimating the average basal area of pine trees. In studying basal areas of similar trees for many years, he has discovered that these measurements (in square inches) are normally distributed with standard deviation approximately 4 square inches.

- If the forester samples $n = 9$ trees, find the probability that the sample mean will be within 2 square inches of the population mean.
- Suppose the forester would like the sample mean to be within 1 square inch of the population mean, with probability 0.90. How many trees must he measure in order to ensure this degree of accuracy?

Problem 3 An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.0 inches and if she randomly samples 200 men, find the probability that the difference between the sample mean and the true population mean will not exceed 0.5 inch.

Problem 4 The downtime per day for a computing facility has mean 4 hours and standard deviation 0.8 hour.

1. Suppose that we want to compute probabilities about the average daily downtime for a period of 30 days.

(a) What assumptions must be true to use the result of Central Limit Theorem to obtain a valid approximation for probabilities about the average daily downtime?

(b) Under the assumptions described in part (a), what is the approximate probability that the average daily downtime for a period of 30 days is between 1 and 5 hours?

2. Under the assumptions described in part (1), what is the approximate probability that the total downtime for a period of 30 days is less than 115 hours?

Problem 5 Suppose that $X_1, \ldots, X_n$ and $Y_1, \ldots Y_n$ are independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Show that the random variable

$$U_n = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2 + \sigma_2^2)/n}} \tag{1}$$

satisfies the conditions of the Central Limit Theorem and thus that the distribution function of U_n converges to a standard normal distribution function as $n \to \infty$.

Problem 6 If Y is a random variable that has an F distribution with ν_1 numerator and ν_2 denominator degrees of freedom, show that $U = 1/Y$ has an F distribution with ν_2 numerator and ν_1 denominator degrees of freedom.

Problem 7 A machine is shut down for repairs if a random sample of 100 items selected from the daily output of the machine reveals at least 15% defectives. (Assume that the daily output is a large number of items.) If on a given day the machine is producing only 10% defective items, what is the probability that it will be shut down? [Hint: Use the 0.5 continuity correction.]

Problem 8 An airline finds that 5% of the persons who make reservations on a certain flight do not show up for the flight. If the airline sells 160 tickets for a flight with only 155 seats, what is the probability that a seat will be available for every person holding a reservation and planning to fly?