Math 464, Theory of Probability **Name:** _____
Fall 2019 **Midterm 1**
Date: 10/10/19 **Time: 75 mins**

- **DO NOT open the exam booklet until you are told to begin. You should write your name at the top and read the instructions.**
- Organize your work, in a reasonably neat and coherent way, in the space provided. If you wish for something to not be graded, please strike it out neatly. I will grade only work on the exam paper, unless you clearly indicate your desire for me to grade work on additional pages.
- You may use any results from class or the text, but you must cite the result you are using. You must prove everything else.
- This exam contains 5 numbered problems. The last sheet is blank. Check to see if any pages are missing. Point values are in parentheses.
- **No books, notes, or electronic devices are allowed.**

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Suppose a family has two children of different ages. We assume that all combinations of boys and girls are equally likely.

   (a) (5 points) Formulate precisely the sample space and probability measure that describes the genders of the two children in the order in which they are born

   (b) (10 points) Suppose we learn that there is a girl in the family. (Precisely: we learn that there is at least one girl.) What is the probability that the other child is a boy?

   (c) (5 points) Suppose we see the parents with a girl, and the parents tell us that this is their younger child. What is the probability that the older child we have not seen is a boy?

2. (a) (10 points) Suppose that $P(A) = 0.4$, $P(B) = 0.3$ and $P\left((A \cup B)^C\right) = 0.42$. Check whether the events A and B are independent or no.

(b) (10 points) Suppose that events A, B and C are mutually independent with $P(A) = 0.3$, $P(B) = 0.4$ and $P(C) = 0.5$ Compute the following probabilities.
$P(A \cap B \cap C^C)$ (ii) $P(A^C \cap B \cap C)$

3. (20 points) The probability of a randomly selected person is suffering from a certain disease is 0.005. The test to detect the disease has a correct detection rate of one, that is, for a person with the disease, the test will always detect the disease. But for a person without the disease, the test has probability 0.002 to give false detection of the disease. Given that a person tests positive for the disease, what is the probability that this person has it?

4. (20 points) Suppose the random variable X is uniform on $[-1, 2]$. What is the probability density function (pdf) of X. Compute the cumulative distribution function (cdf) of X by using its pdf.

5. (20 points) Suppose a continuous random variable X has probability density function (pdf)

$$f(x) = \begin{cases} ax^{-4} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

What must be the value of a? Find $P(-1 < X < 2)$. Find the cumulative distribution function (cdf) of X.