MATH 310 — Quiz 2
20 February 2019

Full name: Solutions

This quiz is worth a total of **20 points**.

This quiz has 2 questions.

1. (10 points) Solve the following homogeneous linear system

$$\begin{array}{rcrcrcrcl} -x_1 & + & 3x_2 & - & 2x_3 & + & x_4 & = & 0 \\ -2x_1 & + & 5x_2 & + & x_3 & - & 2x_4 & = & 0 \\ 3x_1 & - & 8x_2 & + & x_3 & - & 4x_4 & = & 0 \end{array} \tag{1}$$

We start with the augmented matrix

$$\begin{bmatrix} -1 & 3 & -2 & 1 \\ -2 & 5 & 1 & -2 \\ 3 & -8 & 1 & -4 \end{bmatrix} \xrightarrow[R3+3R1]{R2-2R1} \begin{bmatrix} -1 & 3 & -2 & 1 \\ 0 & -1 & 5 & -4 \\ 0 & 1 & -5 & -1 \end{bmatrix} \xrightarrow{R3+R2}$$

$$\begin{bmatrix} \boxed{-1} & 3 & -2 & 1 \\ 0 & \boxed{-1} & 5 & -4 \\ 0 & 0 & 0 & \boxed{-5} \end{bmatrix}$$

Now, by back substitution we get

$-5X_4 = 0 \quad X_4 = 0$

$-X_2 + 5X_3 - 4X_4 = 0 \quad -X_2 + 5X_3 = 0 \quad X_2 = 5X_3$

X_3 is the free variable, let $X_3 = u \quad X_2 = 5u$

$-X_1 + 3X_2 - 2X_3 + X_4 = 0$

$-X_1 + 15u - 2u = 0 \quad X_1 = 13u$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} = \begin{pmatrix} 13u \\ 5u \\ u \\ 0 \end{pmatrix} = u \cdot \begin{pmatrix} 13 \\ 5 \\ 1 \\ 0 \end{pmatrix} \quad u \in \mathbb{R}.$$

2. (10 points) Determine whether the following vectors are linearly independent of dependent

$$\text{(a)} \quad \begin{bmatrix}1\\3\\0\end{bmatrix}, \begin{bmatrix}-2\\-6\\1\end{bmatrix}. \qquad \text{(b)} \quad \begin{bmatrix}4\\2\\6\end{bmatrix}, \begin{bmatrix}2\\1\\4\end{bmatrix}, \begin{bmatrix}6\\3\\9\end{bmatrix}.$$

(a) $c_1 \cdot \begin{bmatrix}1\\3\\0\end{bmatrix} + c_2 \begin{bmatrix}-2\\-6\\1\end{bmatrix} = \begin{bmatrix}c_1 - 2c_2\\3c_1 - 6c_2\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$c_2 = 0 \quad c_1 = 0$ is the only solution, thus $\begin{bmatrix}1\\3\\0\end{bmatrix}$ and $\begin{bmatrix}-2\\-6\\1\end{bmatrix}$

are linearly independent,

(b) $c_1 \begin{bmatrix}4\\2\\6\end{bmatrix} + c_2 \begin{bmatrix}2\\1\\4\end{bmatrix} + c_3 \cdot \begin{bmatrix}6\\3\\9\end{bmatrix} = \begin{bmatrix}4c_1 + 2c_2 + 6c_3\\2c_1 + c_2 + 3c_3\\6c_1 + 4c_2 + 9c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\begin{bmatrix}4 & 2 & 6\\2 & 1 & 3\\6 & 4 & 9\end{bmatrix} \xrightarrow[R3 - \frac{6}{4}R1]{R2 - \frac{1}{2}R1} \begin{bmatrix}4 & 2 & 6\\0 & 0 & 0\\0 & 1 & 0\end{bmatrix}$$

$c_2 = 0$

$4 \cdot c_1 + 2c_1 + 6c_3 = 0 \quad 4c_1 + 6c_3 = 0 \quad c_3 = u \quad c_1 = -\frac{3}{2}u$

$$\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \begin{pmatrix}-\frac{3}{2}u\\0\\u\end{pmatrix} = u \cdot \begin{pmatrix}-\frac{3}{2}\\0\\1\end{pmatrix} \quad u \in \mathbb{R}$$

has infinitely many solutions

thus $\begin{bmatrix}4\\2\\6\end{bmatrix}, \begin{bmatrix}2\\1\\4\end{bmatrix}$ and $\begin{bmatrix}6\\3\\9\end{bmatrix}$ are

linearly dependent.