MATH 310 — Quiz 1
25 January 2019

Full name: ______________________

This quiz is worth a total of **20 points.**

This quiz has 2 questions.

1. (10 points) Use Gaussian elimination to solve the system of linear equations

$$\begin{array}{rcrcrcl} & & 2x_2 & + & x_3 & = & -8 \\ x_1 & - & 2x_2 & - & 3x_3 & = & 0 \\ -\ x_1 & + & x_2 & + & 2x_3 & = & 3. \end{array} \qquad (1)$$

First, we form the augmented matrix:

$$\left(\begin{array}{ccc|c} 0 & 2 & 1 & -8 \\ 1 & -2 & -3 & 0 \\ -1 & 1 & 2 & 3 \end{array}\right).$$

Since $(1,1)$ element is 0, we interchange R1 with R2.

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{array}\right).$$

Next, we eliminate the last nonzero element of the first column by adding the first column to the last one: $R3 + R1$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{array}\right)$$

Next, $R3 + \frac{1}{2} R2$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & 0 & -\frac{1}{2} & -1 \end{array}\right).$$

We have now placed the system in upper triangular form.

By back substitution, we get $-\frac{1}{2}X_3 = -1$, $X_3 = 2$

$2X_2 + X_3 = -8$, $X_2 = -5$ and $X_1 - 2X_2 - 3X_3 = 0$, thus

$X_1 = -10 + 6 = -4$.

$$\boxed{X_1 = -4, \ X_2 = -5, \ X_3 = 2}$$

2. (10 points) Find the inverse of each of the following matrices, if possible, by applying the Gauss–Jordan Method.

$$\text{(a)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

(a) Note, that $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is an elementary matrix of type 1. Thus, to find its inverse we just negate the only non zero off-diagonal entry. So $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

(b) We start the Gauss-Jordan elimination process by ~~the~~ constructing its augmented matrix

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

First, we interchange R1 with R2

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \xmapsto{R3 - R1} \left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 & -1 & 1 \end{array}\right) \xmapsto{R3 + 3R_2}$$

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 6 & 3 & -1 & 1 \end{array}\right) \xmapsto[\frac{1}{6} \cdot R_3]{\frac{1}{2} \cdot R_1} \left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \end{array}\right) \xmapsto[R1 - \frac{1}{2}R3]{R2 - 2R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{3}{2} & 0 & -\frac{1}{4} & \frac{7}{12} & -\frac{1}{12} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \end{array}\right) \xmapsto{R_1 - \frac{3}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{4} & \frac{1}{12} & \frac{5}{12} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \end{array}\right)$$

Thus, the inverse is

$$\begin{pmatrix} -\frac{1}{4} & \frac{1}{12} & \frac{5}{12} \\ 0 & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{2} & -\frac{1}{6} & \frac{1}{6} \end{pmatrix}.$$